The extraordinary ascent of gold prices in recent years has attracted more economically sophisticated analysts to gold than have studied it for quite a while. Gold, the stuff of legend and myth, evokes more emotion than any other commodity, but it isn’t exempt from rational economic analysis. Earlier this year, Claude B. Erb and Campbell R. Harvey completed a thorough, critical review of the various propositions that have been put forth to justify investing in gold. This paper was ably summarized by Michael Edesess in a previous Advisor Perspectives article.

Another paper, by George Patterson and Lingjie Ma, which appeared a few months earlier, addressed some of the same propositions but has received less attention. Besides building an econometric model of gold prices, Patterson and Ma were also attempting, very practically, to answer the question, “Is gold overpriced?” The two pairs of authors were unaware at the time that they were setting off down the same road. They reached largely similar conclusions, even though Patterson and Ma applied a seldom-used analytical method to get at the answer to their question, and Erb and Harvey were not trying to answer that specific question.

Patterson, without Ma, presented a summary of their work recently to a gathering in Boston. Both Patterson and Ma work in quantitative investments at BMO Global Asset Management, and Patterson holds a doctorate in physics, as well as the CFA and CFP® charters.

Patterson explained that the motivation for the paper arose directly from their professional work. Their employer, Bank of Montréal, often flies them to Toronto, where they repeatedly saw how Canadians were caught up in the recent commodities boom culturally as well as economically. Even as the U.S. was suffering economically, Canada seemed to Patterson and Ma to be filled with “gold bugs.” Half of the Toronto stock (TSX) index, Patterson pointed out, is in energy and metals, and the metals allocation includes many gold mining companies, which helps to explain the Canadian cultural attention to gold.

Patterson and Ma were perplexed at hearing analysts talk of gold moving to $3,000 or $5,000 a troy ounce. When gold hit $1,900 an ounce in September 2011, Patterson told Ma that he wanted to short it. Ma replied that they should first build an analytical model, and so the paper was born. Patterson said that when Ma summarized their research in China, the Chinese audience loved it, but the Canadians hated even the premise that there could be any decline in the price of gold.


**The economics of gold mining**

Patterson ticked off some pertinent facts about gold: Almost all the gold that has ever been mined is still available, and there are few industrial uses for the metal apart from its application to circuit boards. Gold’s chief use is social, not industrial. Gold jewelry is especially important in India and, increasingly, in China. Ma visited Canadian mines in the course of his research. The extraction process, Patterson said, has not changed significantly in quite a while. It takes a long time to develop a new mine, because gold is usually found in comparatively inaccessible places. Gold mining is also sensitive to environmental regulation, because the industrial process of separating gold from the ore requires arsenic. These problems contribute to its scarcity.

Contrary to the popular image of prospectors panning for gold nuggets, he said, gold mining entails the removal of vast quantities of earth to a processing facility. A ton of metal-bearing earth from a typical mine yields between one and 10 grams of gold dust – though 10 is considered extraordinary. The quantities within the earth are so diffuse as to be invisible, and locating the gold-bearing ore requires sophisticated industrial analysis. The usual yield is a few grams. But the yield of even one gram of gold can be profitable, depending upon the price of gold at the time. Most of the reserves currently being developed are of lower quality than in the past, meaning that yields are declining. There are some reserves of higher quality, but they’re located in areas of extreme environmental sensitivity, which would raise the financial cost of extraction.

Many of the most productive mines are in areas with considerable political risk. Analysts told Patterson and Ma that if the price of gold were to decline below $1,100, some mines would shut down. But the extraction companies are large firms that have diversified across different metals and have assets in politically stable countries, so they would not necessarily be badly hurt by a fall in the price of gold.

Patterson briefly reviewed gold’s price history over the last couple of hundred years, pointing out that there have been price spikes at times of crisis, such as the American Civil War. He added that the common statements that gold protects against inflation and against a weak dollar are not obvious from casual observation of the time series of prices. This disconnect between commonly cited wisdom and past performance was a motivation for his and Ma’s and Erb and Harvey’s research.

Patterson and Ma began their research by asking broker-dealers and others how they forecasted the price of gold. These dealers and analysts had models based on supply and demand and on sentiment, but these were not econometric models, and their forecasts were very widely dispersed. All the same, Patterson said, there is an upward bias in the consensus estimates. They supplied some of the data series that Patterson and Ma needed. These discussions also defined and narrowed Patterson and Ma’s choices of explanatory variables for their model.
Building a model to explain the price of gold

Patterson and Ma wanted as long a run of data as they could get, so they used U.S. data, including gold's price history, starting in 1968. Most financial economists would build a model by identifying variables that might explain the price of gold, and then perform a standard regression analysis. Patterson and Ma chose a novel approach, using quantile regression, a technique that was introduced by Koenker and Bassett in 1978.³

Patterson and Ma attempted to be parsimonious in their choice of explanatory factors. They chose the price of West Texas Intermediate crude oil, the unemployment rate, the Dow Jones Industrial Analysis, the Treasury-bill rate, the Consumer Price Index, GDP and an index of the U.S. dollar. (The choice of the DJIA rather than the S&P 500 was a practical one based on the licensing cost of the data for publication.) Patterson admitted, however, that they didn’t winnow the factors -- they merely used all that they supposed might be of interest. The only possible explanatory factor that they considered and chose to omit (though Patterson did not say why) was the U.S. money supply.

Patterson distinguished quantile regression, which he and Ma used, from ordinary least squares (OLS) analysis, which every beginning student of statistics learns. In OLS regression, one finds the typical or average response of the dependent variable to a change in the independent variables. Quantile regression allows the analyst to study the response of the dependent variable at different levels of its range of values, including extreme levels, to changes in the independent variables. Patterson said that quantile regression has seldom been used in finance, where the data tend to be very “noisy,” but that it is used in ecological modeling, crime studies and health-care analysis. The example he gave was of low birth weight. Usually, the medical researchers are less interested in the response of the average baby to various medical interventions than they are of babies of very low weight to such interventions. Quantile regression has also been applied to real-estate analysis, in which the researchers might want to understand the response of very high-priced houses, rather than the typical house, to changes in the economic environment.


Ma studied with Koenker, one of the inventors of quantile regression, and was therefore a persuasive advocate of this tool. Patterson cautioned that, for those trained in mainstream regression analysis, comfort with the concepts of quantile regression comes very slowly. He asserted that one advantage of quantile regression over traditional OLS for segmented data series is that quantile regression uses the entire distribution of values of the dependent variables, even while it shifts the focus of the analysis to those quantiles that are of concern. The choice of using price levels, rather than price changes (or returns) was Ma’s, also. The choice could be justified by a close examination of the correlations of the variables. Price levels also allowed a more direct answer to the question that was the title of their paper. Still, some tests suggested that the results of their model would not have been very different if based on changes.

To set the mathematical scene, Patterson displayed a table of correlations of the price of gold with his
and Ma’s choice of explanatory variables. These correlations were calculated using OLS regression.

<table>
<thead>
<tr>
<th>Correlation with Gold Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil Price</td>
</tr>
<tr>
<td>Unemployment</td>
</tr>
<tr>
<td>Dow-Jones Industrial Average</td>
</tr>
<tr>
<td>T-Bill Rate</td>
</tr>
<tr>
<td>Inflation (CPI)</td>
</tr>
<tr>
<td>GDP</td>
</tr>
<tr>
<td>U.S. Dollar Index</td>
</tr>
</tbody>
</table>

We see here that some of these correlations, like the level of the stock market and the T-bill rate, are very low. The low values are economically and statistically insignificant. Nonetheless, he and Ma retained all these variables in their construction of a model.

Patterson presented, as an example of the usefulness of quantile regression, the relationship between the level of unemployment and the price of gold. He and Ma had chosen the unemployment rate along with GDP as explanatory variables because they jointly represent the overall state of the economy. Of Patterson and Ma’s seven explanatory variables, the unemployment rate had the most effect on gold prices in both OLS and quantile regressions. (They found that GDP also had a large influence on the price of gold.) The OLS estimate showed that a 1% increase in the unemployment rate could be expected to result in a $90 per ounce increase in the price of gold. The quantile regression, however, showed that at low or median price levels, a 1% increase in the unemployment rate could be expected to produce a $20 increase in price, whereas at high prices, the same increase in the unemployment rate could be expected to produce a $180 increase in price.

The price of oil is also one of the leading determinants of the price of gold, probably because gold extraction is so energy intensive. Patterson said that those who trade gold frequently worry most about production costs, and the price of oil in particular. Yet, he said, commentators are more likely to talk of gold as an inflation hedge or a currency hedge. Like Erb and Harvey, Patterson and Ma found no relationship between the price of gold and inflation when they considered the entire time span of their analysis. But Patterson and Ma broke the inflation series into two periods, pre-1982 and post-1981. They hypothesized that gold would hedge against inflation only if inflation appeared to be a real threat to the economy, as it did in the U.S. in the late 1970s and through 1981. It turned out that the data did support this hypothesis: Gold appeared to be an inflation hedge in the U.S. before 1981 but not after. (Erb and Harvey observed that whatever relationship appeared to exist between gold and inflation was
explained by one year, 1980.) Patterson and Ma also found little support for the traditional assumption that gold is a hedge against a weak dollar. (Erb and Harvey dismissed this relationship, too, because the real price of gold has historically shifted in unison across the many currencies that they studied.)

After performing a univariate quantile regression of the price of gold against each of their chosen explanatory variables, Patterson and Ma constructed a multivariate quantile regression model using all of these variables together. Having built this model with data up to 2008, they then tested it, out of sample, against the price of gold subsequently. On the whole, the model was fairly stable and worked surprisingly well, especially when considering the extreme values of the price of gold in the test period.

**Further questions**

Patterson said that, for all the talk of the importance of supply and demand in India and China, these quantities are very difficult to measure, so they were left out of the model. An added difficulty is that the wealth to be found in China and India today is relatively new, and the pertinent data series, even if they existed, would be too short. Still, it would not be interesting to explore this train of thought, he said. For example, gold jewelry is most in demand in India during the wedding season, which could lead to seasonal price swings. Also, some analysts have speculated that China may soon reach a stage of economic development where the demand for gold jewelry, and consequently the price of gold, will dramatically spike.

There are also analysts who claim that the supply and demand of gold by central banks can fully explain the price of gold, but again, Patterson and Ma could not find data series that would have allowed them to test this hypothesis. And there is the familiar claim, alluded to earlier, that gold is a hedge against a major political crisis. But as there have been only a few of these over the last couple of hundred years and quantifying them is very difficult, Patterson and Ma saw no way to test this claim, either.

Patterson added that, having built a model to predict the price of gold, he’d like to turn it around to see if it can use the price of gold to interpret the state of the economy today. He and Ma did not, however, pursue this train of thought in their published paper or in the evening’s presentation.

Patterson and Ma’s purpose wasn’t to create a forecasting tool as much as to understand the sensitivities of the price of gold. Quantile regression can help to explain the sensitivities of gold at today’s prices, rather than, as OLS regression does, what the sensitivities historically have been. Patterson was asked if their model would have looked much different if it had been constructed 10 years earlier, when prices were much lower. All the quantiles would still have existed, but the actual values within the quantiles would have been different. Patterson replied that he thought that the model would have looked much the same, except that inflation might have appeared to play a somewhat larger role.

Patterson concluded that their model is currently (as of Sept. 17, 2013, the date of the talk) indicating that the price of gold is fairly reasonable. His personal intuition is that over the next year, the price of gold will come down slowly and will “test the lows again” at around $1,200, unless we are struck by a severe and unexpected severe political crisis.
Adam Jared Apt, CFA, is a financial advisor and the owner of Peabody River Asset Management, based in Cambridge, MA.