



The How, Why, and Why Not of “What If?” The Use of Numerical Simulation in Private Client Investment Management

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July 15, 2008

The complexity of planning and managing investments for private clients has led to increasing use of numerical simulation methods within the financial community. There are a variety of areas where techniques such as Monte Carlo simulations have become increasingly popular. Among these are asset-liability and spending policy analysis for retirement planning, stress testing in risk assessment, tax deferral strategies and “portfolio opportunity distribution” methods in performance measurement. Numerical methods are also employed to address parameter uncertainty in portfolio optimization exercises and to improve the quality of statistical analysis of markets when higher moments are observable in data samples.

While numerical methods are growing in popularity, we should remind everyone of the big limitation of numerical simulations: Our conception of “what if?” is wholly dependent on experience to date. *We can’t simulate events we haven’t considered.* It should also be remembered that numerical methods are most suitable for situations wherein the analytical problem is too complex to allow tractable algebraic solution. Alternative approaches are also available for consideration of this class of problem.

Most simulation methods are based on the “Monte Carlo” approach where we assume the parameters of the probability distribution of a variable, and then take a series of random draws from that probability distribution. The assumed distribution can be simple (e.g. normal) and have independent and identically-distributed (IID) time series properties, where each member has the same probability distribution and the members are mutually independent. Or it can have complex features such as skew, kurtosis, time-varying parameters, and even jumps. We can also have systems of multiple correlated random variables and take random “vector” draws. The downside is that if there are lots of moving pieces (variables), the number of draws needed to get a sufficient sample is large and hence time consuming. It is tempting to overly simplify the problem to cut computation times

An alternative to Monte Carlo techniques is “bootstrap” methods. Bootstrap simulations are based on real data samples rather than the parameters of probability distributions. We’ve only lived through the path of history that actually happened. *What else might have happened even if the distribution of possible events was similar?* Let’s look at an example:



- Imagine you have 180 monthly observations in a return time series for asset class X
- Pick a random number R from 1 to 180. Think of numbers selected in a “bingo” game
- Make the “Rth” observation from the data series, the first observation in a new series
- Repeat the random pick another 179 times, each time adding the Rth observation from the original series to the new series

The order will be different in the new series and some original observations might be omitted, while others may appear more than once, so parameters like mean and standard deviation of data series will change. Using bootstrap methods, you can create as much simulated data as you want to explore “what might have happened”.

The most popular use of numerical simulations in private client investment management is for asset allocation decisions within 401K and other retirement plans. Firms such as Financial Engines and Invesra use numerical simulations to find the asset mix that has the highest likelihood of meeting retirement goals. However, these analyses are often criticized as insufficient. Often the analysis uses only historical asset class return information, which is certainly not an admissible estimate of future return distributions. Such methods also do not consider the extent to which we’ll miss our goals if we do miss. Considering only probability and not magnitude of failure is called a Lower Partial Moment 0 approach. Most importantly, these analyses have the implicit assumption that the investor’s tolerance for risk is constant, irrespective of wealth, age or past performance

A different view of the investor life cycle suggests that the key issue in formulating investment policies is how aggressive or conservative an investor should be at each moment in time to maximize their long term wealth subject to a shortfall constraint (a floor on wealth). This approach was originally developed by Jarrod Wilcox (“Harry Markowitz and the Discretionary Wealth Hypothesis”, *Journal of Portfolio Management*, Spring 2003) and is recommended by CFA Institute. One way to express this concept is

$$U = E\{R * (1-T^*) - L S^2 (1-T^*)^2 / 2 \}$$

- L is the ratio of total assets/net worth
- T* is the effective tax rate
- Total assets and liabilities on an investor’s “life balance sheet” can be flexibly defined to include the present value of implied assets such as



lifetime employment savings, expected expenses such as college tuition, insurance, estate taxes

Taxation, particularly “lot by lot” capital gain taxation is very complex to model, so numerical methods are used to simulate potential after-tax outcomes of strategies. In a 2008 research [study](#), Jarrod Wilcox uses Monte Carlo simulation to estimate the proper adjustments in asset allocation, allowing for uncertainty about future capital gain tax rates and the life span of the investor. In related [work](#), diBartolomeo simulates the extent to which opportunities for tax deferral are related to the cross-sectional dispersion returns of securities within an asset class.

Another popular application of numerical simulation is portfolio optimization under parameter uncertainty. Markowitz efficient frontier analysis assumes that we *know with exactness* the parameters of the return probability distributions (mean, standard deviation, correlation). In reality, we only have estimates of the future parameters, so we’re underestimating the uncertainty of the problem. **We are missing the risk of being wrong.** One way to address parameter uncertainty is “resampled” optimization, wherein we repeatedly draw new parameter estimates from a simulation and calculate the optimal portfolio for each draw. We can then take some kind of average of the many portfolios that proved optimal for a specific instance of data outcomes. Early work based on bootstrapping was: Bey, Roger, Richard Burgess and Peyton Cook, “Measurement of Estimation Risk in Markowitz Portfolios”, University of Tulsa Working Paper, 1990. Later Richard Michaud published an approach using multivariate Monte Carlo methods in his 1998 book, Efficient Asset Management (published by Harvard University Press, Cambridge, MA). A variety of other techniques have been developed to address parameter uncertainty in portfolio formation including Bayesian methods, and “robust” optimization.

When we make asset allocation and other investment decisions, these choices are based on estimates of the parameters of return distributions. Statistical estimation of return parameters like mean, standard deviation, correlation, beta, etc. can be very biased if the data is not normally distributed and IID. One approach to estimation in the presence of higher moments is Markov Chain Monte Carlo (MCMC) analysis. In MCMC, we combine Monte Carlo simulations with Bayesian statistics to get more robust parameter estimates. [Sfridis](#) (2006), is a recommended resource.

Monte Carlo simulations of “what could go wrong” called “stress testing” are a typical approach to evaluating the “downside risk” of portfolios. The most common downside risk measure is “Value-at-Risk” (VaR) analysis. Investors should be cautioned that VaR is an incoherent measure of risk in that there are some investment situations where VaR will lead investors to clearly incorrect



conclusions about the relative risks of different investments. Downside risk methods are most suitable for financial entities with high degrees of leverage where non-survival is plausible

Studies have also shown that consideration of “downside risk” is relevant only in a limited number of investment situations. One such study is: Cremers, Jan, Mark Kritzman and Sebastien Page. “Portfolio Formation with Higher Moments and Plausible Utility”, Revere Street Working Paper Series 272-12, November. 2003.

Private client portfolios often have peculiarities that make it less meaningful to measure their performance against conventional benchmark indices. Examples are different tax rates, legacy positions, SRI constraints, and differences in investor concern about absolute losses as compared to benchmark-relative underperformance. One alternative is to do Monte Carlo simulations of the possible range of returns to an actively managed portfolio given the same universe and constraint set, as put forward by Surz, Ronald J. in "Portfolio Opportunity Distributions: An Innovation In Performance Evaluation," Journal of Investing, 1994, v3(2), 36-41. If a portfolio under evaluation ranks highly within the range of simulated possible return outcomes, that's a positive outcome that can be evaluated for statistical significance much more quickly than traditional performance measures.

If you wish to pursue “hands-on” use of simulation methods, a good summary of numerical procedures used for investment analysis can be found at website www.investment-analytics.com. There are also “quasi” Monte Carlo methods that allow us to get a good sample of complex distributions with fewer random draws. A summary can be found in [Li and Winker](#) (2000).

Monte Carlo simulation and related numerical methods are useful tools for dealing with the complexity of real world investing circumstances, but are not a substitute for an understanding of the investor needs and preferences. Simulation methods can be put to good use in testing alternative formulations of closed form analytical solutions.

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