How to Calculate Your Personal Safe Withdrawal Rate

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July 6, 2010

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There is a wonderful folk theorem in the world of investment advice that says an annual withdrawal amount of roughly 4% of the initial balance of a retirement account can last an investor almost indefinitely. One source of this idea is Jonathan Guyton, who says:

> *"Typically, the safe initial withdrawal rate for pretax portfolios is around 4% when all the equities are U.S. large-cap stocks; when smaller-cap stocks are introduced, usually the safe initial withdrawal rate increases modestly to about 4.5%."*

Guyton's finding and other research on this topic rely on many assumptions, some of which may not hold in today's environment.

Instead, I provide a new, transparent analysis of the safe withdrawal rate (SWR), one that enables investors to explicitly update their SWR based on new beliefs about their future returns and inflation. Historical data are not required.

Overview of current SWR work

Once the initial balance of a pre-tax retirement account is established, its hypothetical investor can set an annual spending goal for a time horizon of his choosing. The investor withdraws an amount of money equal to the initial balance times his initial withdrawal rate, an amount that hopefully suffices for the investor's purposes. The problem he and his advisor face is to find the maximum withdrawal rate (called SWR) possible when the time horizon is unforeseeable or indefinite, as most retirements are.

Under this traditional approach, both returns and inflation determine how account balances evolve. My approach, however, is independent of historical returns and inflation. Rather, we determine the SWR based on various future states of the world as defined by the investor's subjective probability distributions of returns and inflation.

Previous research has used historical market data, assumptions about portfolio allocations (e.g., 60% stock, 40% bonds) intended to achieve sufficient returns, and a variety of simulations and back tests to calculate the account's yearly balances for a given withdrawal rate. Then the largest withdrawal rate for a sufficiently long time horizon is chosen as the SWR, given the chosen portfolio allocations.

Put briefly, most current research analyzes SWR for back-tested, hypothetical portfolios. Essentially, researchers calculate the SWR for an investor by requiring him to assume their future balances will evolve according to historical return rates and inflation. (See, for example, Cooley, Korn, and Considine. A summary review can be read on Bogleheads.)

A clear problem with these SWR approaches is the possibility that the near future will present substantially larger return volatility. Global deleveraging, residual fear from recent market panics, massive government spending and the possibility of unanticipated inflation all make added volatility highly likely.

How many investors or advisors would want to use historical portfolio performance data to design long-term portfolios under these conditions?I think not many, but that is just what current approaches to the SWR problem do.

How can we account for other beliefs?

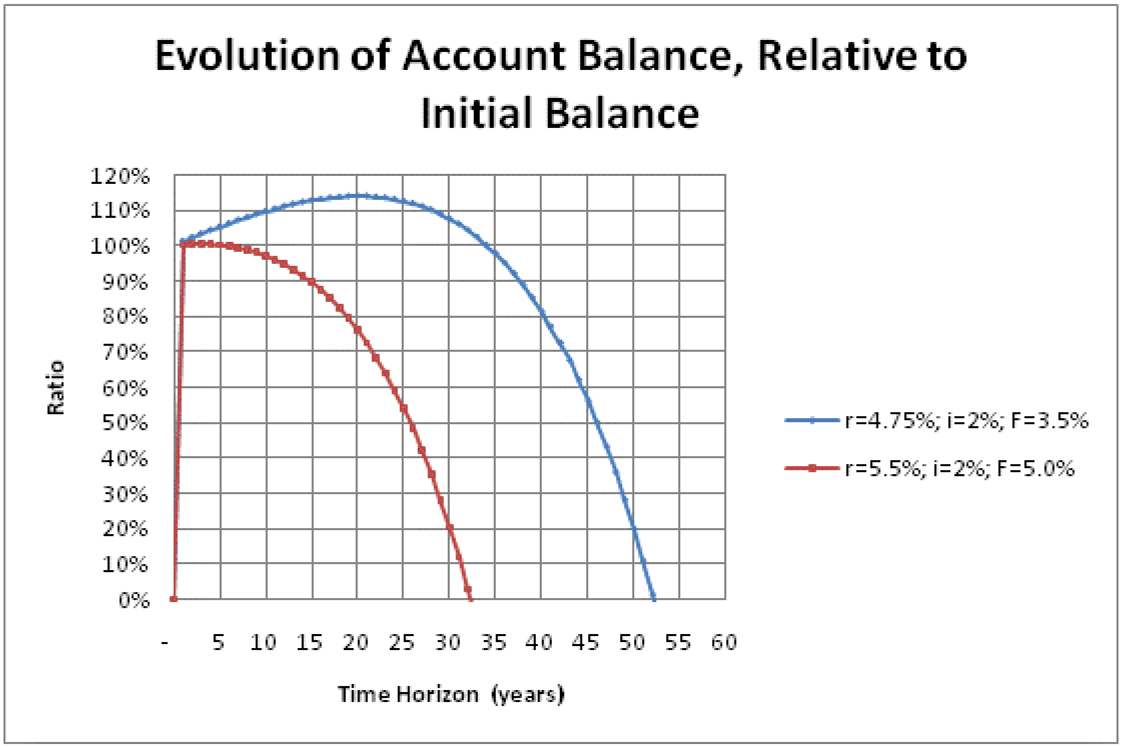
I offer a new approach: Let the investor incorporate his beliefs, however they are formed, about the evolution of his own future returns, and let him define the inflation levels he expects. The investor can explicitly capture those beliefs about the future (which beliefs always drive his decisions anyway) to produce a personalized result for the SWR. This approach requires a model, since it doesn't use back-testing. My model is built around a "closed form" formula for SWR as the basis to incorporate investor beliefs.

Account withdrawal mechanics

First, let's be clear about the investment account's cash flows. The initial withdrawal amount is adjusted annually based on the rate of inflation; it would be constant if inflation were zero. Also, at the end of each period, the remaining balance grows based on the returns earned on its investments.

Figure 1 below shows how the account balance changes over time under two scenarios: a return of 4.75%, inflation of 2%, and a withdrawal rate of 3.5% in one case; and a return of 5.5%, inflation of 2%, and a withdrawal rate of 5% in the other.

Figure 1: Example of Normalized Balances



The inflation rate is the same for both curves, but account balances diminish more rapidly with the higher return rate. Because the withdrawal rate is greater for the high-return case, the account is drawn down faster than it grows, compared to the low-return situation with a smaller withdrawal rate.

Safe Withdrawal Rate: An "optimum" value

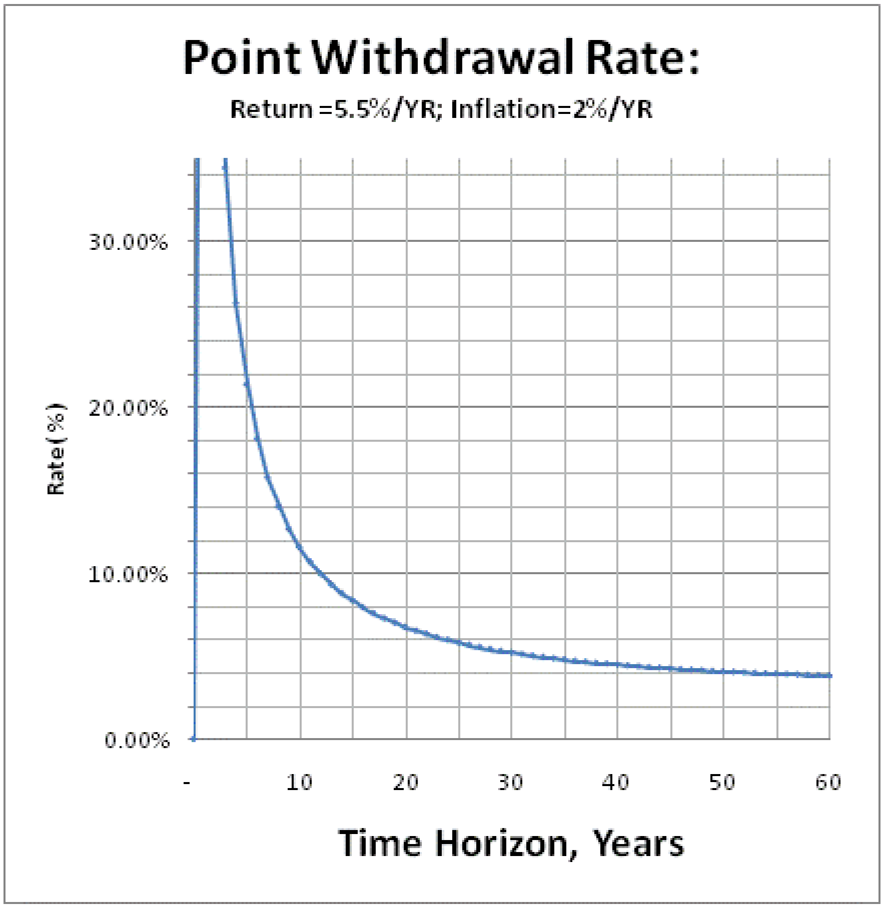
There must be an optimum SWR value. If the SWR is small enough, then the balances remain positive for as long as we like, but in that case our withdrawal amounts likely will be insufficient to support our lifestyle. As the SWR grows, one's lifestyle may be better supported, but the balances will drop to zero over ever-shorter time horizons. So we seek a SWR that is a useful compromise. The main problem is that future returns and inflation are unpredictable.

Let's begin to solve the problem by finding the maximum withdrawal rate, such that the account value is zero at the end of the time horizon, assuming the returns earned on investments and inflation are known. I call this rate the Point Withdrawal Fraction, because we still haven't addressed the variability of returns and inflation.

Any series of returns and inflation can be replaced by an equivalent constant return or an inflation rate for the whole horizon. I lose no generality by assuming that return rates and inflation rates are constant.

The Point Withdrawal Fraction is illustrated in Figure 2.

Figure 2: Point Withdrawal Fraction



This graph is valid for the specific returns and rate of inflation assumed. It plots the withdrawal rate against the arbitrary number of years N in the time horizon.

How can we deal with the randomness of returns and inflation?

The innovation: Apply your own beliefs

I account for random future returns and inflation by capturing the investor's personal beliefs about his investment returns and inflation. These beliefs dominate all decisions, whether the investor accepts "historical" values, bets on his own, or combines both. The investor can use this framework to make his own best guesses based on his subjective and specific *beliefs*.

This model structures the assumptions each investor inherently makes and captures them in a useful way. The investor has no need to embrace any particular historical interpretations or predictions based on them, although I recognize that all predictions implicitly rely upon possibilities revealed by history. I simply define a range of possible values of return rates and inflation rates and then specify a probability distribution for those values. Each distribution, by its probability weighting, defines a *scenario*. A pair of return rate and inflation rate scenarios, as defined by their distributions, combine to form a *path*.

These paths derive from different beliefs about the rates that will occur. Each path describes a belief about a future state of the world that produces a specific pair of return and inflation probability distributions. The issue here is treating the randomness of those assumptions and not their constancy. A path encapsulates an entire future "world," as represented by the joint probability distribution of returns and inflation.

Examples of beliefs

To illustrate this idea, assume five possible values for the future returns:

{0%, 2.5%, 5%, 7.5%, 10%}.

Now assume a set of values for inflation. In the model we can choose any number and any values of return rates and inflation rates. Here, I use five values for inflation

{2%, 4%, 6%, 8%, 10%}

I capture the beliefs about the paths under different conditions by assigning subjective probabilities that each scenario value actually occurs over the time horizon. Each particular path, comprised by definition of two scenarios, has a total in this example of 5x5 possible pairs of fixed return rates and inflation rates.

To consider a wider view of our future, we choose three return rate scenarios ("Bad Luck," "Skill," and "Good Luck") and three inflation scenarios ("Minimal," "Observable," and "Runaway"). Then there are nine possible paths, each defined by the specified pair of scenario distributions.

The following six graphs show the investor-generated, subjective probability distributions for return rates (Figure 3) and inflation rates (Figure 4) for all time horizons of interest. Remember, these probabilities capture an investor's current beliefs about his future performance, for his own idiosyncratic reasons.

The investor can explicitly analyze extremely rare but costly return scenarios simply by including a very negative return value (e.g., -70%) and assigning that value a probability as part of a scenario distribution. Of course, such a negative return would rapidly deplete the account, but it also would occur with very low probability.

Figure 3: Three return rate scenarios

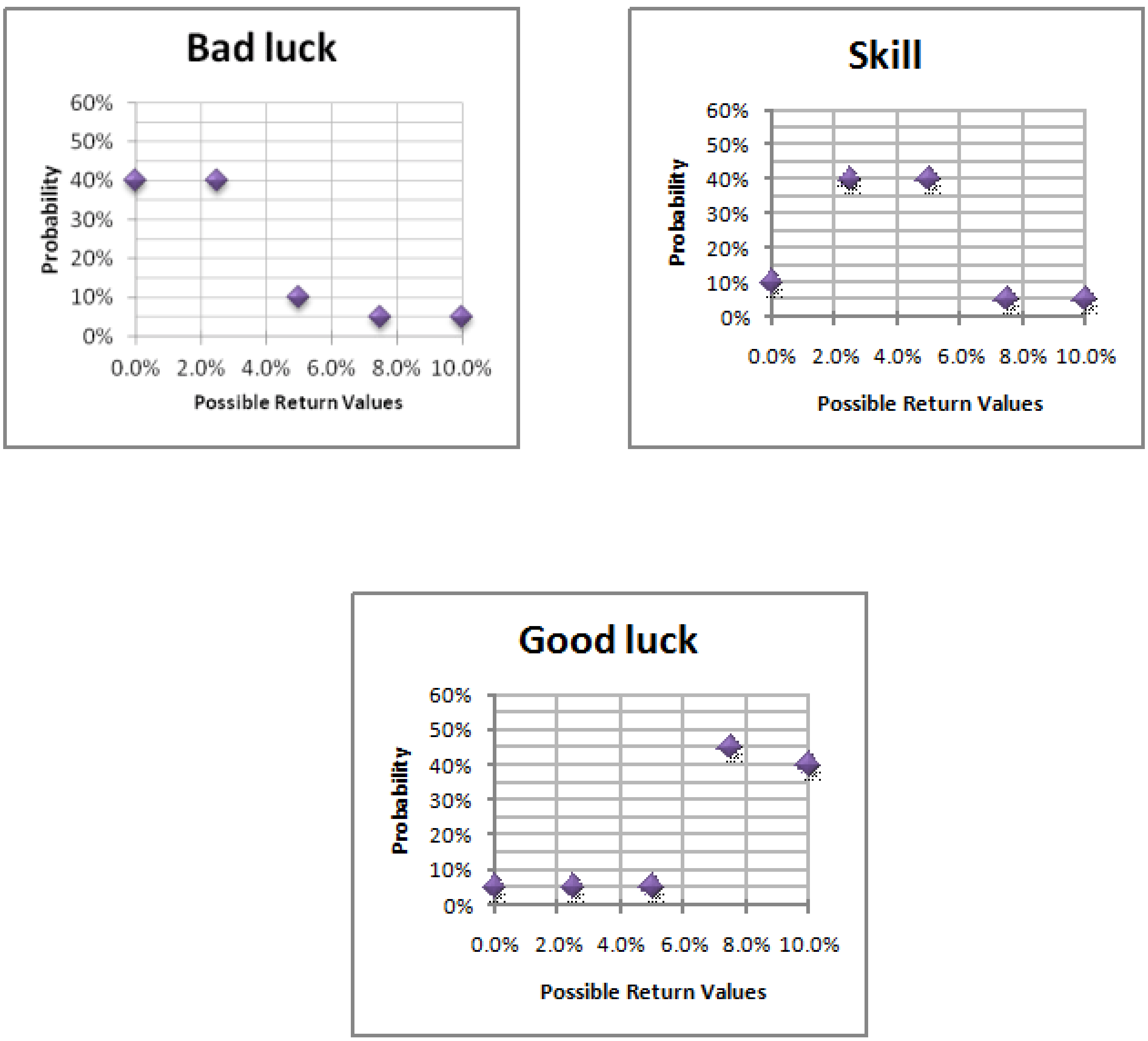
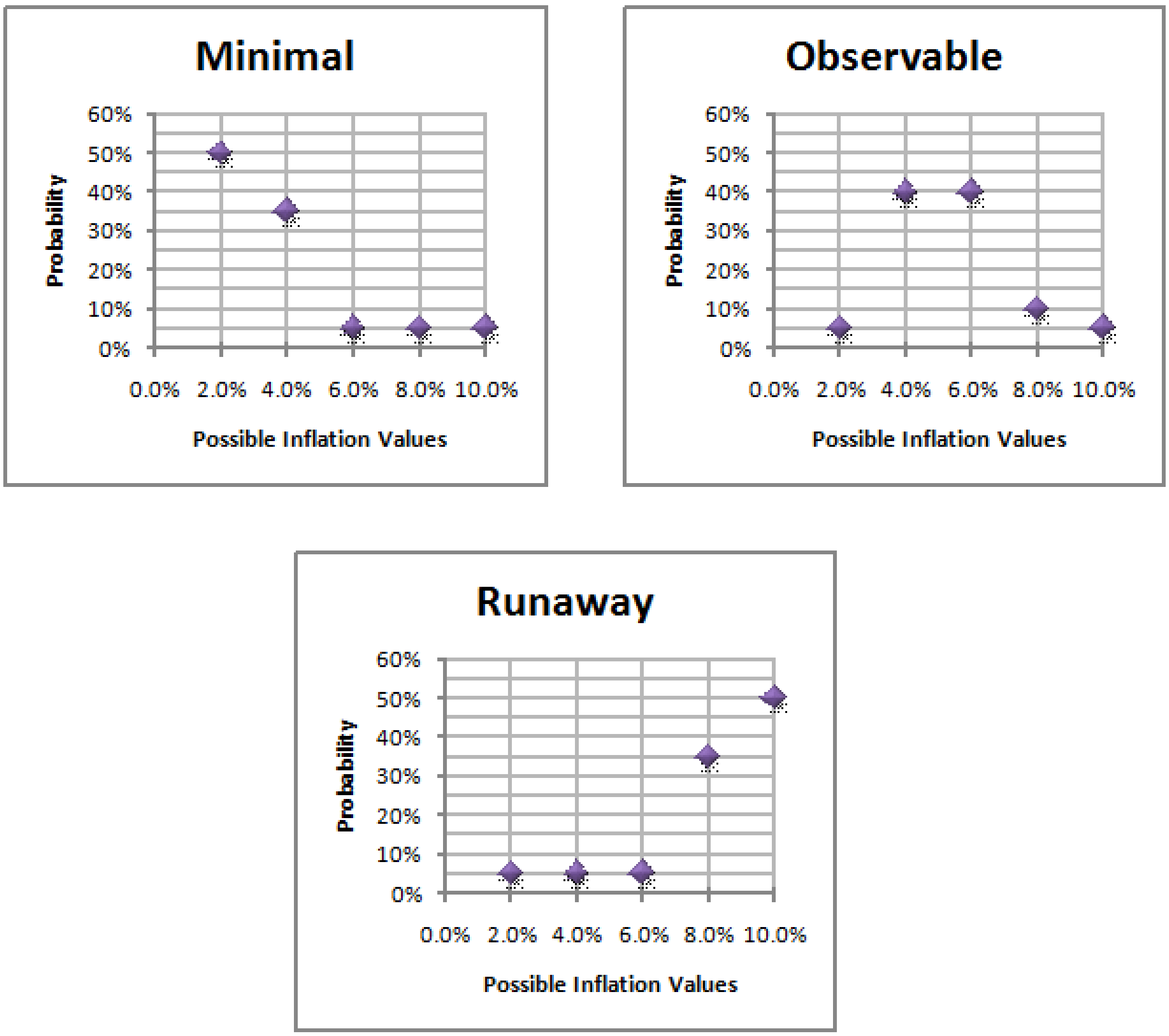


Figure 4: Three inflation rate scenarios

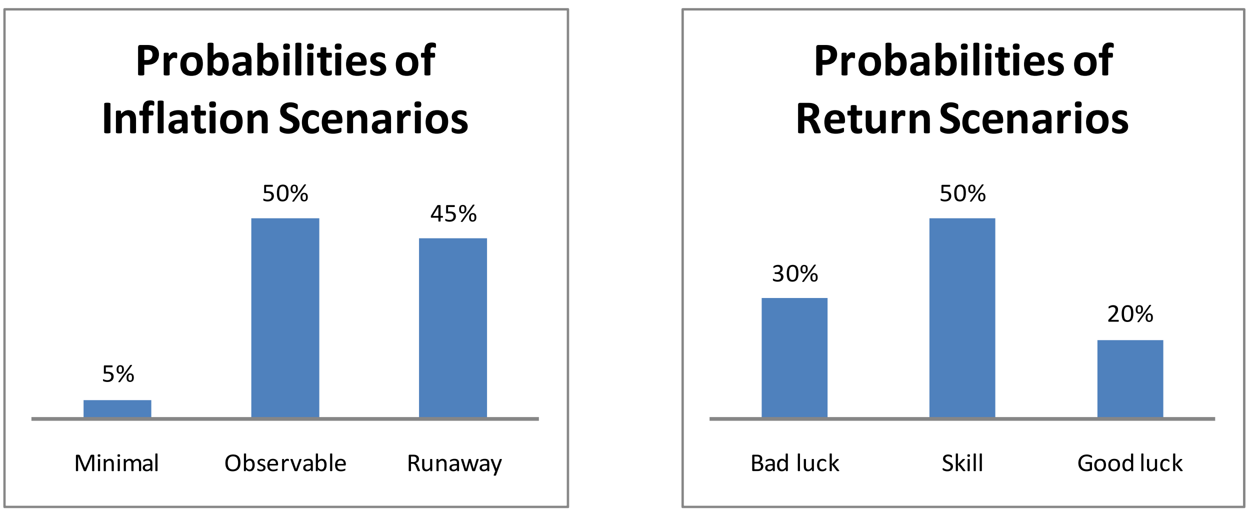


I assume independent probability distributions for returns and inflation over the time horizon. This assumption has some empirical support in Dexia Asset Management's Special Report, "Inflation and equity returns: what's the link?" (March 2010), which says:

> *"Our general conclusion is that it is extremely hard to define hard-and-fast rules with respect to the link between inflation and equity returns."*

The investor also must specify his subjective probabilities that each scenario will be realized. Thus, he produces two other distributions like those in Figure 5. The matrix in Figure 5 shows the joint probabilities for the nine paths, assuming returns and inflation are independent.

Figure 5: Scenario Probabilities



Joint Probabilities of paths (independent scenarios)

			Inflation		
			Minimal	Observable	Runaway
			5%	50%	45%
Return	Bad luck	30%	1.500%	15.000%	13.500%
	Skill	50%	2.500%	25.000%	22.500%
	Good luck	20%	1.000%	10.000%	9.000%

Sensitivity of SWR to beliefs

Armed with this set of beliefs and assumptions, I can draw some conclusions about the mean SWR. Figure 6 shows the mean SWR versus horizon-years curve for each of the nine paths. These curves reveal the strong sensitivity to different paths (recall that a path is a pair of return rate and inflation rate probability distributions that generate the future evolution of the account balance).

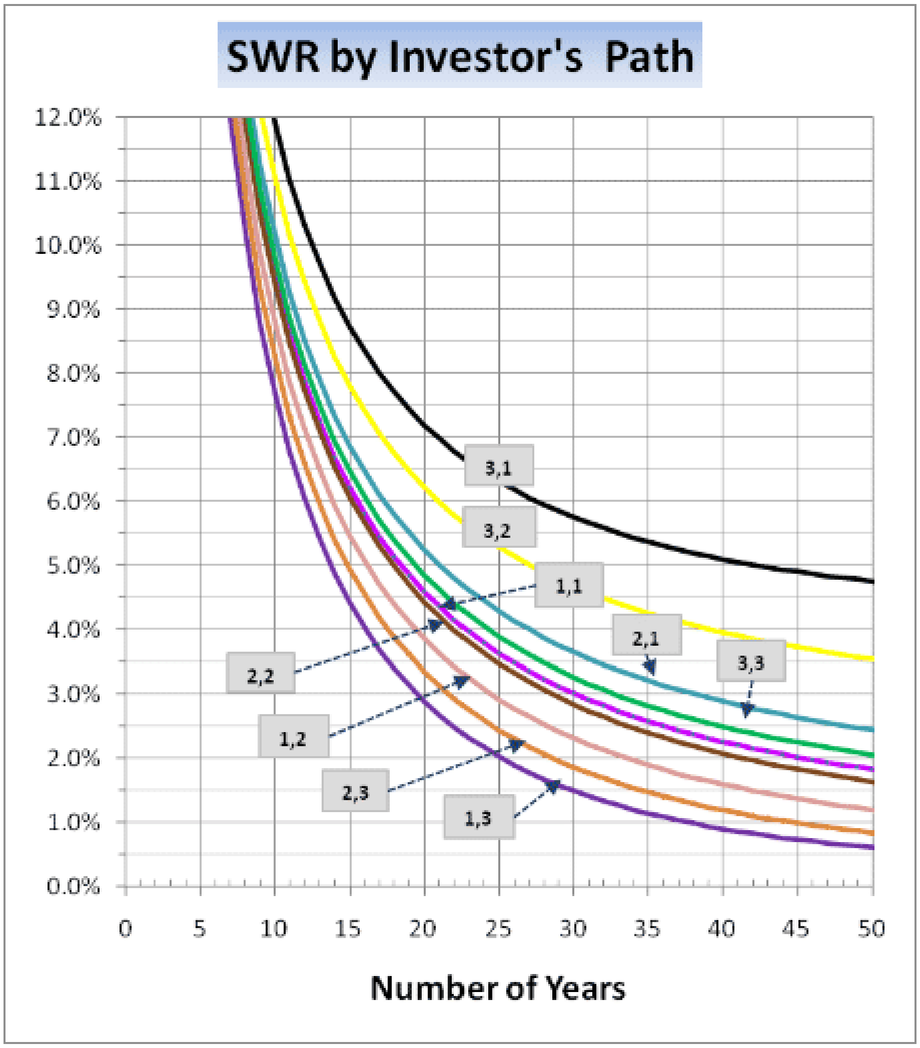
The matrix below the curves shows the assumed probabilities of the return and inflation scenarios, thereby determining the probability of each path. The cells in the matrix correspond to the labels on the graph. For example, the highest curve in the graph is for path (3,1), which is return scenario 3 “Good Luck”, inflation scenario 1 “Minimal”. For any horizon N, this path among the nine gives the largest mean SWR. The lowest mean SWR curve results from path (1,3), which is the path (Bad Luck Returns, Runaway Inflation).

The ranking of the curves is shown in the matrix in Figure 6. The rankings are not always obvious. Consider which curve should be higher: that of good luck with returns and observable inflation (3,2) or skillful returns and minimal inflation (2,1)? The calculations reveal the former is superior to the latter.

A SWR of 4% can be viable for time horizons ranging from 16 to over 50 years, depending on which path is realized.

I can now simplify these results by taking expectations over the probabilities we assign to the occurrence of the paths themselves.

Figure 6: Mean SWR for each path



Ranking of Path Results

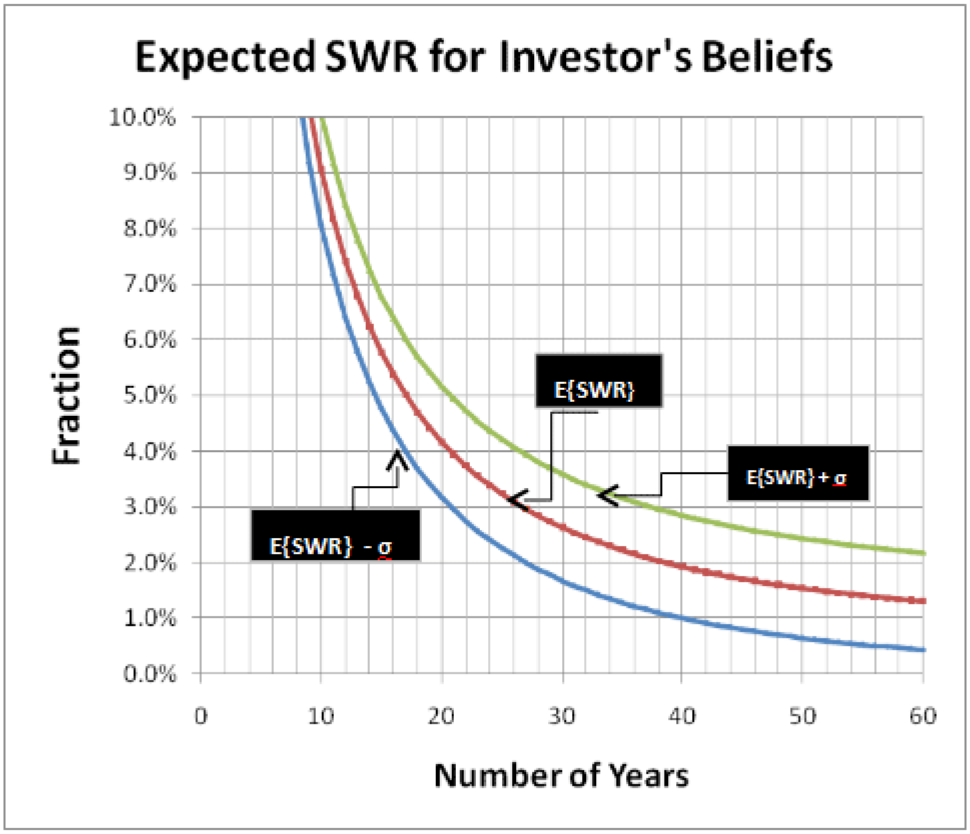
			Inflation		
			Minimal	Observable	Runaway
			5%	50%	45%
Return	Bad luck	30%	5	7	9
	Skill	50%	3	6	8
	Good luck	20%	1	2	4

Figure 7 smooths the nine outcomes according to our beliefs about each path's probability of occurrence. If the beliefs change, so will the graph. The middle curve is the expected value of SWR, and the two other curves show the mean and the standard deviation around the mean. Keep in mind that the mean and standard deviation SWR curves are generated by the particular probability distributions chosen by the investor. Different distributions produce different results.

For these beliefs, a horizon of 20 years has a SWR of 4.2% ± 1% and 30 years requires a SWR of 2.6% ± 1%.

These results illustrate the sensitivity of a SWR to the assumptions. Investors should use this approach to understand the required returns for a desired SWR and the constraints on a SWR that his beliefs induce.

Figure 7: SWR as function of investor beliefs



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