Fundamental Indexing: Breakthrough or Old Idea in New Marketing Garb?

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For a related discussion of fundamental indexing, see Michael Edesess' article in this issue, [Fundamental Indexing: A Verbal Optical Illusion](), and our article, [Fundamental Indexing Debunked](), which appeared on August 5, 2008.

Advocates of "fundamental indexing" claim that capitalization-weighted indexes overweight overpriced stocks and underweight underpriced stocks. Therefore, they argue, a portfolio created using weightings that are independent of market prices will have less overpricing bias than one created using market weightings. This paper examines their claims and finds them to be provably false, given the advocates' own assumptions. Therefore, their argument cannot provide support for any particular portfolio strategy.

Several respected names in the investment field have lined up behind a concept they call "fundamental indexing". In its typical implementation (though not necessarily the only implementation), fundamental indexing takes the form of a discipline for value-tilting a stock portfolio. Its proponents make larger claims for its superiority however, beyond the well-known fact that value and small-cap stocks have historically outperformed the broad stock market (Fama and French 1992, Arnott et al 2005).

Efforts to gain wide acceptance for fundamental indexing have resembled a well-designed marketing campaign, combining two words, both having a certain heft —"fundamental" and "index"—with an intuitively plausible postulate: "the cap-weighted market portfolio overweights overpriced stocks and underweights underpriced stocks."

Is there substance behind the arguments for fundamental indexing, or is there only marketing buzz? In this paper I attempt to pin down the claims made for the fundamental indexing concept, and investigate whether the claims can be substantiated.

The search for alpha and the Schwert rule

The historical record showing that value and small-cap stocks have outperformed market averages has prompted money managers who specialize in these areas —or wish to—and who seek to promise "alpha" to suggest that this record will continue. But historical records are notoriously poor predictors in the investment field. Historical patterns of anomalous risk-adjusted market-beating performance tend to obey the Schwert rule: "After they are documented and analyzed in the academic literature, anomalies often seem to disappear, reverse, or attenuate." (Schwert 2003.) Hence, justifications not based on the historical record are needed to present a strong argument that some particular style of investing will produce alpha in the future.

The search for a justification has led several advocates of value and small-cap philosophies to advance the argument that it is in the nature of market-cap-weighted indexes to "overweight overpriced stocks and underweight underpriced stocks." Several advocates have attempted mathematical proofs of this conjecture (Arnott and Hsu 2008, Hsu 2006, Treynor 2005). Both Kaplan (2008) and Perold (2007) have pointed out, however, that some of the proofs implicitly assume that the person allocating investment weights has knowledge of the "fair" values of the investments—in direct contradiction to the advocates' assumption that fair values are unknown.

The fundamental indexing approach

The approach of the fundamental indexing advocates ("FI advocates") involves first making estimates of the fair values of stocks based on a fundamentalist methodology (which for the FI advocates is a value-tilted methodology, based on factors such as low price-to-dividend ratio). They then weight a portfolio in the same way it would have been weighted in a market index if the market prices had been equal to the fundamental price estimates. This could be characterized as a "fair-value-weighted portfolio," where fair values are estimated according to a fundamental estimation methodology, as contrasted with the market-cap-weighted portfolio in which fair values are estimated by the market prices.

Pinning down the claims

Many claims have been made for the fair-value-weighted portfolio, and against the market-cap-weighted portfolio (Arnott and Hsu 2008, Hsu 2006, Arnott *et al* 2007, Treynor 2005), though they are not easy to pin down. Among the various statements, the most complete set of claims against the market-cap-weighted portfolio and in favor of alternatively-weighted portfolios is the following (RAFI 2008 p. 4):

> "The return drag from capitalization weighting—overweighting overpriced securities and underweighting underpriced securities—is a structural long-term return inhibitor. … [T]he goal of price indifferent indexing is to randomize portfolio weights to approximately allocate half of our money to overvalued shares and half to the undervalued.
>
> "We know that capitalization weighting will structurally place more in securities whose stocks are priced above fair value and less in those that are priced below fair value. Why? Because the weights relative to fair value are not random; they are linked to price and the errors embedded within that price!"

An effort to obtain specific and testable claims from this passage yields the following:

(1)A market-cap-weighted portfolio will be overpriced on average—that is, its weighted average price will be greater than the weighted average of the stocks' fair values—though some of the stocks in it may be overpriced and some underpriced.

(2)For a portfolio to have zero expected pricing error it must use weightings that are not based on market prices, even if the weightings have to be chosen by a random process.

It should be noted that the FI advocates assume a quantity called "fair value" is meaningful, though it is unknown and ill-defined. "Fair values" are unobservable even after the fact. Fair values are stocks' "(unknowable) discounted future cash flows" (Arnott *et al* 2005). The fair value of a stock is the best estimate at any point in time of its future cash flows, discounted by the best estimate at that point in time of the future discount rate. It is *not* the present value of future *realized* (and therefore observable)—i.e. *ex post*—earnings discounted by the future *realized* discount rate, because the future realized values are not the same as, and may bear little relation to, the best estimates of these values *ex ante*. Hence, it is difficult if not impossible to test hypotheses about fair values using observable data.

Nevertheless, it is possible to evaluate the claims of the FI advocates, because they base their claims on an assumed mathematical relationship between fair values and market prices.

Assumptions made by FI advocates

FI advocates base their claims on the following assumptions:
(a)Each stock has, at any point in time, an unknown fair value.

(b)Each stock has a known market price which is an estimate of the stock's fair value, and is therefore equal to the fair value plus an unknown estimation error (often called "noise").[i] The estimation error is the overpricing (underpricing) of the stock relative to fair value.
(c)The estimation error is uncorrelated with fair value, and its expected value is zero.

These seem like mild assumptions but in fact assumption (c) is quite strong. It assumes that market estimates of fair value are not biased either for or against a particular stock or group of stocks. It also assumes that market estimates of fair value are not all biased in one direction or another. Many would agree now that both of the foregoing assumptions could be in doubt in view of the "bubble" of the late 1990s to early 2000. These are far from the only assumptions that could be made. It could be assumed, for example—with different implications—that the estimation error is uncorrelated with market price instead of being uncorrelated with fair value.

Are portfolio weights that are uncorrelated with prices better than market weights?

One of the arguments made for fundamental indexing—and even for randomly-selected stock weights—is as follows. As long as one posits that a fair value exists, not that it is known, then there is a positive cross-sectional correlation between cap weights and estimation error. (This, of course, depends on the acceptance of assumption (c) above.) This induces the drag that is avoided by choosing weights in a fashion such that there is no cross-sectional correlation at a point in time. Fundamental weights, if chosen without regard to price, could have a zero cross-sectional correlation with estimation error. Random weights would have a zero cross-sectional correlation (depending on how "random" is defined and how the weights are chosen).[ii]

The problem with this argument is that for every weighting scheme that is uncorrelated with market prices, there is a complementary weighting scheme that can co-exist with it in the marketplace and that is also uncorrelated with market prices.[iii] Together the two weighting schemes—both uncorrelated with market prices—if universally adopted, would comprise the market portfolio. Therefore they cannot both be superior to a market-cap-weighted portfolio, even on a risk-adjusted basis.

Can the claims be proven true or false?

Can claims (1) and (2) be proved true or false given assumptions (a) through (c)? The answer is an unequivocal "Yes." The claims can be conclusively proved false. It can be proved, under the assumptions (a) through (c) made by the FI advocates themselves, that the market-cap-weighted portfolio does not misprice stocks on average. The Appendix presents a mathematical proof. I will now present a non-mathematical argument to show intuitively why it is true.

Instead of dealing with stocks and prices initially, let us address a slightly different problem with the same mathematical structure. Suppose a random global sample of adults is taken, and their heights are measured using a measuring technique that is only approximate, but at least its errors are unbiased—that is, the measurements are as likely to err on the high side as on the low side. Suppose the average height of all adults in the world is, say, 5' 6". Let the average of the heights measured for the sample also be 5' 6".

What is the best estimate of the average measurement error for this sample? If we believed the average measurement error was, say, positive, then we would have to believe the true average height of the sample was less than 5' 6", and if we believed the average measurement error was negative then we would have to believe the true average height of the sample was greater than 5' 6". Why would we choose one over the other when we know the measurement technique is unbiased, and that the average measurement came out the same as the population average? Therefore, the best estimate of the average measurement error for this sample is zero.

Suppose someone told you that the 5' 6" measured sample average was an overestimate because the sample average "overweights overestimated heights and underweights underestimated heights." Since the sample average is height-weighted (it is an average of heights) the 5' 6" measure is likely to be an overestimate. This might cause you some momentary confusion, but it should be only temporary. (FI advocates seem to be under the misimpression that the person's height—or the stock's price—is somehow weighted twice, as if it were an average of squared heights, or prices, being taken, not of heights or prices themselves.) Fortunately for those still confused, the Appendix's proof confirms that 5' 6" would indeed be an unbiased estimate.

Now let's go back to stocks. Suppose the average capitalization of all stocks is a billion dollars. Since the average market "mispricing" is zero (by the assumption stated earlier that market price is an unbiased estimate of fair value), a billion dollars is also the average fair value of all companies. Now suppose a sample of these stocks is taken, forming a portfolio; and suppose that the average capitalization of the stocks in the portfolio is also one billion dollars. What would we expect to be the average "mispricing" error in this portfolio? Under the assumptions above, the mispricing error is unbiased. There is no reason to believe the average of the "correct" prices of the stocks in the sample is either less than one billion dollars or greater than one billion dollars—especially given that the sample average is the same as the population average—and therefore the best estimate of the portfolio's average mispricing is zero.

A market-cap-weighted index portfolio has the same average market cap as the average market cap for all stocks—which by assumption is the same as the average fair value for all stocks. Therefore, there is no reason to believe that the average mispricing in a market-cap-weighted portfolio is anything other than zero.

What if a portfolio's average market cap is not equal to the overall average market cap?

Let's go back to the height question. Suppose the average height in the random sample was not measured to be the same as the population average—5' 6"—but greater than that, 5' 8". Now what would we estimate to be the average measurement error? Interestingly, we should now estimate that the average measurement error is positive. (This is proved in the Appendix.) Similarly, if a randomly-selected stock portfolio has an average capitalization that is greater than the average market cap, then—under the assumptions stated above—the average mispricing would be expected to be positive. And if its average capitalization is less than the average market cap, then the average mispricing would be expected to be negative.

Is this an argument for a "structural" tendency for the marketplace to overprice large-cap stocks and underprice small-cap stocks—and therefore an argument that large-cap index portfolios (larger than average market-cap) overweight overpriced stocks? No, it merely exposes the fragility of the underlying assumptions, (a) through (c).

The fact that a mathematical proof of a conjecture about stock prices can be presented based on a set of assumptions renders the conjecture no less vulnerable to the Schwert rule than the record of past outperformance of small and value stocks. Once it becomes widely known—or believed—that stock portfolios that overweight large-cap stocks are overpriced and portfolios that underweight large-cap stocks are underpriced, there will be (and already has been) a rush to invest in small cap stocks that will drive their prices up at the expense of large-cap stocks. In the somewhat mystical language of assumptions (a) through (c), this would mean that the market price estimator of "fair value" would become biased upward in the case of small stocks and biased downward in the case of large-cap stocks, thus invalidating the assumptions.

It is certainly not inconceivable that a time could come when larger-capitalization and growth stocks are underpriced and smaller-capitalization and value stocks are overpriced. This could come about, for example, if the bandwagon for investing in small and value stocks takes off in a big way, producing a value- and small-stock bubble. That time could well be now, and we wouldn't know it since fair values are assumed to be unknown. If such market conditions are possible—and they surely are—then there cannot possibly be a valid proof of the general claim that a market-price-weighted index portfolio overweights overpriced stocks.

The Appendix result does, however, lead to several interesting observations.

The non-persistence of randomness

First, it should be noted that the result applies to *random* samples whose sample means depart from the population mean. If the same individuals in a sample that was taken at random at one time are later measured again, it is no longer a random sample. For example, suppose the sample of people whose average height is 5' 8"—two inches above the population mean—is later measured again. If its average does not "regress to the mean" but is again 5' 8", we would this time believe that it is not an overestimate, even if the first time we might have expected that a portion of the two-inch overage was due to overestimation. We would *not* react to the second 5' 8" measurement by saying, "Well, if we just wait and keep measuring it is bound to regress to the mean eventually."

Similarly, suppose a group of stocks selected at random have greater-than-average market caps, and we conclude that they are likely to be overpriced relative to fair value. (This conclusion applies *only* under assumption (c), and *only* if the sample is random.) Now suppose that they are re-measured shortly thereafter (one month, or one quarter, or one day after), and that their "mis"-pricing has been found not to have reverted toward the mean (which would be evidenced by their not having realized a higher rate of return than the market average). We should then conclude *not* that their mispricing hasn't been corrected yet, but rather that they were not mispriced in the first place.

Biased estimation

Second, issues raised by results similar to the one in the Appendix are known in the field of statistics, and can be dealt with by using biased estimators (Judge and Bock 1983). While doubts have always been raised about the rationality of market prices, most people would concede that the market has a certain collective intelligence. Therefore, it should be capable of using a biased estimator of "fair value"—as a statistician might—if that strategem helps to correct for the possibility that a high estimate is an overestimate. If the market does use a biased estimator of fair value however, that would invalidate one of the key assumptions of FI advocates—that the market price is an unbiased estimate of fair value.

This last observation raises an intriguing possibility. If the market consistently uses a biased estimator—lowballing high prices and nudging up low estimates in order to correct for the possibility that high prices in random samples tend to be partly overestimates, and low prices partly underestimates—then would that not mean that high-cap stocks will tend to be *under*priced (their market prices are biased estimates of their fair values), and therefore an investor could take advantage of this fact? No, because the estimator will *always* be biased (unless a stock's market cap falls below the average market cap)—and therefore the price will never revert to an unbiased estimate of fair value. Hence, no excess return could be realized. Doesn't this reasoning also raise the question of whether certain types of stocks—if they really were "structurally" overpriced (or underpriced) as the FI advocates claim—might never revert to a "correct" price, so as to allow fundamental indexers to realize their anticipated excess return?

Fair values and angels

Before we go too far with these discussions so that they begin to resemble debates about how many angels can stand on the head of a pin, let us reiterate that we are dealing with questions about unknown "fair values". Neither fair values, nor angels, are observable (except perhaps by the enlightened few). Suffice it to say that none of this supports the arguments of FI advocates.

In fact, arguments for the "structural" superiority of value and small stock strategies, upon close examination, reduce to nothing more than a claim of a strategy with the ability to discern fair values, and therefore to detect undervalued stocks better than other observers. This type of strategy has a name: it is an active stock-picking strategy. The "fundamental indexing" philosophy is, in fact, no more than an old strategy in a new marketing wrapper.

Conclusion

The debate over fundamental indexing has centered on whether it is merely a value-tilted, active investment philosophy, or whether (as FI advocates claim) it is a passive, quantitative strategy that takes advantage of a statistical property of stock prices causing market-cap-weighted indexes to be overpriced on average. This paper has shown conclusively that the FI advocates' argument that a market-cap-weighted index overweights overpriced securities and underweights underpriced securities is without merit. Therefore, their fundamental indexing strategy can lay no claim to an ability to outperform the market benchmark, beyond the claims that have already been made over the years by numerous others on behalf of value-oriented investment strategies.

Appendix A

I begin by deriving a general result for two independent normal random variables X and Y with respective means $\mu_X = 0$ and μ_Y, and respective variances σ_X^2 and σ_Y^2. Let $Z = X + Y$. Then $\mu_Z = \mu_X + \mu_Y$ and $\sigma_Z^2 = \sigma_X^2 + \sigma_Y^2$. What is the conditional expectation of X given that $Z = z$?

It follows from Bayes's Theorem that the conditional cumulative distribution function of X is

$$\begin{aligned} F_{X|}(x) &= \Pr(X < x \mid Z = z) = \Pr(X < x \mid X + Y = z) \\ &= \Pr(X < x \text{ and } X + Y = z) / \Pr(X + Y = z) \end{aligned}$$

and convoluting X and Y, the right side equals

$$\int_{u=-\infty}^{x} p_X(u)p_Y(z-u)du \Big/ \int_{u=-\infty}^{\infty} p_X(u)p_Y(z-u)du$$

where p_X is the normal density function with mean $\mu_X = 0$ and variance σ_X^2 and p_Y is the normal density function with mean μ_Y and variance σ_Y^2.

The probability density function $f_{X|}(x)$ of X conditioned on $Z = z$ is the derivative of $F_{X|}(x)$ with respect to x, or

$$f_{X|}(x) = p_X(x)p_Y(z-x) \Big/ \int_{u=-\infty}^{\infty} p_X(u)p_Y(z-u)du$$

and the expected value is

$$E(X \mid Z = z) = \int_{x=-\infty}^{\infty} x\, f_{X|}(x)\, dx = \int_{x=-\infty}^{\infty} x p_X(x)p_Y(z-x)dx \Big/ \int_{u=-\infty}^{\infty} p_X(u)p_Y(z-u)du\,.$$

Inserting formulas for the normal densities p^X and p^Y and performing a series of standard formula manipulations yields

$$E(X \mid Z = z) = (z - \mu_Z)\,\sigma_X^2/\sigma_Z^2. \qquad (1)$$

If $z = \mu_Z$, this reduces to

$$E(X \mid Z = \mu_Z) = 0.$$

Now let us apply this result to the portfolio question posed above, beginning with the assumptions (a) through (c) posited by advocates of fundamental indexing—namely, that each stock's market price (i.e., capitalization) is given by P_i and that the company has an unknown fair value V_i which is related to P_i by $P_i = V_i + U_i$, where V_i and U_i are independent and the expected value μ_U of U_i is zero.

Let μ_p be the average market price, i.e., capitalization, of all stocks *1* through *N* in the market index. I will derive the expected portfolio mispricing for a specified portfolio, given that the average capitalization of the stocks in the portfolio is equal to the average market cap μ_p.

First form elementary portfolios of *n* randomly selected stocks containing either all or none of each stock *1* through *N*. Define a portfolio's underpricing or overpricing $\overline{U}$ as the average amount by which the market price of the stocks in the portfolio lies above or below the average fair value.

Thus,

$$\overline{U} = \left(\sum_{k=1}^{n} P_{i_k} - \sum_{k=1}^{n} V_{i_k} \right) \Big/ n = \overline{P} - \overline{V}$$

where $\{i_1,\ldots,i_n\}$ is a subset of $\{1,\ldots,N\}$.

What is the expected value of the error $\overline{U}$ given that the average capitalization of stocks in the portfolio $\overline{P}$ equals μ_p, the average market capitalization? The question is essentially the same as the one posed above about $E(X \mid X + Y = \mu_Z)$. If *n* is large enough that $\overline{P}, \overline{U}$ and $\overline{V}$ are approximately normal, then

$$E(\overline{U} \mid \overline{P} = \mu_p) = E(\overline{U} \mid \overline{U} + \overline{V} = \mu_p) = \mu_U = 0.$$

This result has been derived only for elementary portfolios containing either all or none of each stock. To derive an equivalent result for any portfolio, note that any portfolio can be expressed as a linear combination of elementary portfolios. Therefore its pricing error $\overline{U}$ is $\sum a_i \overline{U}_i$ where $\overline{U}_i$ is the average error of the *i*'th elementary portfolio and the a_i are constants. Then

$$E(\overline{U} \mid \overline{P} = \mu_p) = E(\sum a_i \overline{U}_i \mid \overline{P} = \mu_p) = 0 .$$

This implies the same conclusion as for elementary portfolios that the portfolio's expected overpricing/underpricing is zero if the average capitalization of its constituent stocks is equal to the average capitalization of all stocks.

Thus, if the average capitalization of stocks in a portfolio ($\overline{P}$) is equal to the average capitalization of all stocks in the index (μ_p)—as is the case for a market-cap-weighted index portfolio—then the expected portfolio mispricing is zero. The market-cap-weighted portfolio is on average neither overpriced nor underpriced. This contradicts the claims of fundamental indexing advocates that market-cap-weighted portfolios overweight overpriced stocks and underweight underpriced stocks.

Note from equation (1) that under the given assumptions, if the average capitalization of stocks in the portfolio is greater than μ_p then the expected mispricing will be positive, while if the average capitalization of stocks in the portfolio is less than μ_p the expected mispricing will be negative.

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[i] The assumption is often stated in a slightly different multiplicative form in which the known market price P is equal to V times a factor $(1 + \varepsilon)$.

[ii] I owe the suggestion of this formulation to personal correspondence from Bob Ferguson, 8/15/2008.

[iii] Let P_i be the total market price (i.e., market cap) of the i'th stock. Given a weighting scheme $\{W_i\}$, where W_i is the weighting of the i'th stock, let A be the aggregate dollars invested in this scheme, and let $A_i \leq P_i$ be the amount invested in the i'th stock. Define stock i's complementary weighting as $W_i^* = \left(P_i - W_i A \Big/ \sum_{j=1}^{N} W_j \right) \times \sum_{j=1}^{N} P_j \Big/ \left(\sum_{j=1}^{N} P_j - A \right)$. Then the aggregate amount invested in the complementary weighting is $A^* = \sum_{j=1}^{N} P_j - A$. If the $\{W_i\}$ are uncorrelated with fair values $\{V_i\}$ then so are the $\{W_i^*\}$. Together, the two comprise the market portfolio.